Networks in Space

Spillovers in Amazon Deforestation

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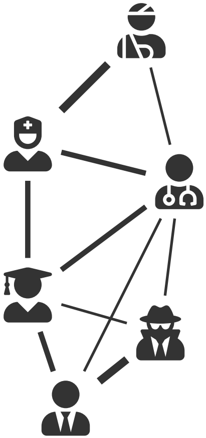
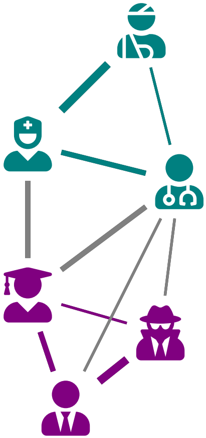
Motivation

- *Economic agents* are **embedded in networks** that give rise to **spillover effects**, but
  - we *do not observe* underlying networks.

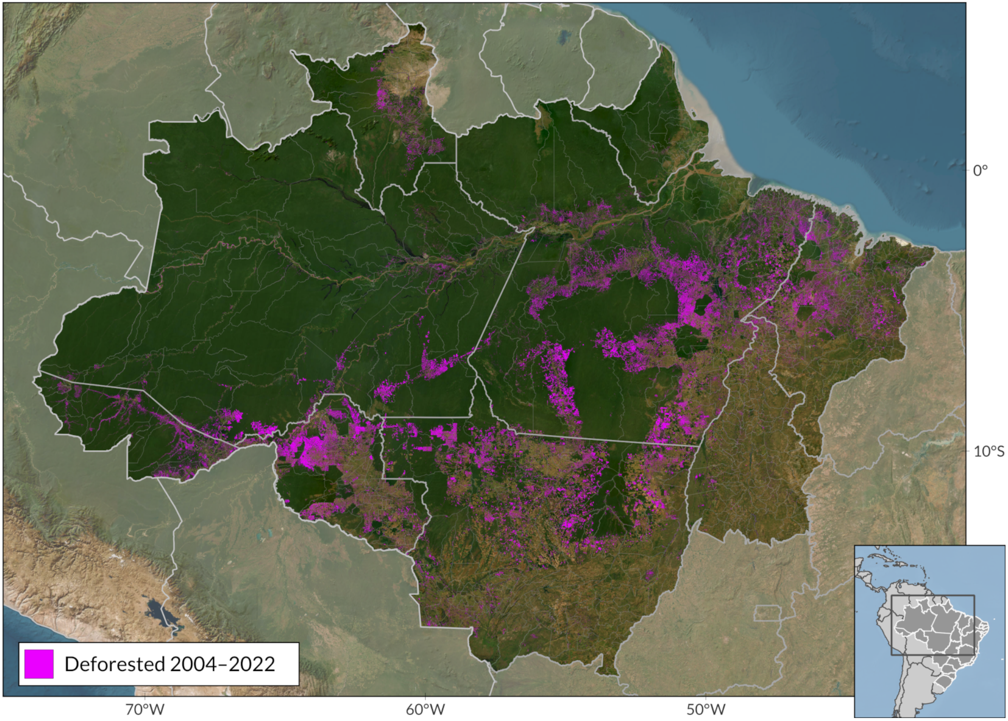


Motivation

- *Economic agents* are **embedded in networks** that give rise to **spillover effects**, but
  - we *do not observe* underlying networks.
- **Amazon deforestation**[a] is coined by spillovers.
  - threatening the *climate*, biodiversity, ...



[a] And countless other and related issues; see Alfaro-Ureña et al., 2022; Assunção and Rocha, 2019; Canen et al., 2023; Dhyne et al., 2021; Giovanni et al., 2022; Richards, 2017; Souza-Rodrigues, 2019; Vom Lehn and Winberry, 2022.


0°
10°S
70°W
60°W
50°W
Deforested 2004–2022

Research question

1. How can we **learn more about spillovers** using *available information*?
2. What is the *role of spillovers* in targeted **deforestation interventions**?

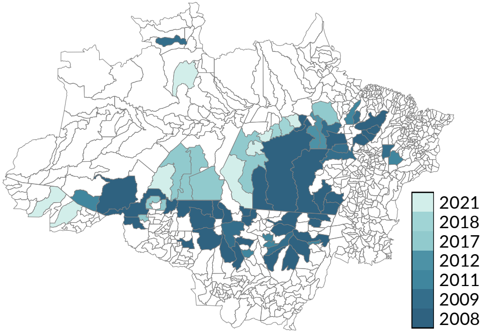


Figure 1: Priority municipalities in the *Brazilian Legal Amazon* and their date of designation.

Approach

- **Framework:** joint model of **spillovers & latent networks**
  - *deforestation model* with *peer effects*
  - *latent space* network model
- **Estimation:** flexible shrinkage for **data-driven insights**
  - nuanced priors to use *external information*
  - *full posterior inference* via sampling
- **Application:** deforestation in the **Brazilian Amazon**
  - *priority municipalities* focus on *hotspots*
  - positive or negative spillovers, multiplier

Findings

- **Joint model** — more accurate & deeper insights
- **Priority municipalities** — prevent deforestation
  - ≈65 km^2 prevented per year[1]
  - *strategic complementarities* complicate inference

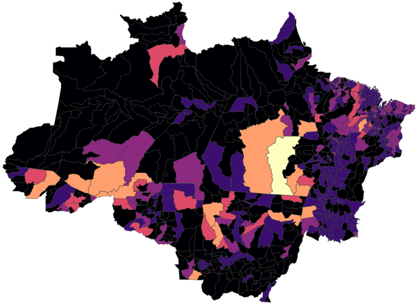


Figure 2: Relative strength of outward spillovers (out-degree centrality) as recovered from a flexible network model.

[1] Compared to estimates of 20 without and 25 km^2 with contiguity spillovers.

Literature & Contributions

- **Deforestation** — environmental economics[2]
  - + *theoretical foundation* for deforestation spillovers
  - + evidence for positive *spillovers to be utilized*, and *endogenous spillovers*
- **Spillovers** — spatial & network econometrics[3]
  - + *bridge* literatures via aggregation & latent space
  - + *flexible* model that exploits available information

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[2] Alix-Garcia and Gibbs, 2017; Amin et al., 2019; Araujo et al., 2023; Arima et al., 2014; Assunção and Rocha, 2019; Balboni et al., 2023; Burgess et al., 2023; Busch et al., 2015; Cisneros et al., 2015; de Andrade, 2016; Giljum et al., 2022; Gollnow et al., 2018; Harding et al., 2021; Kuschnig et al., 2023; Roitman et al., 2018; Richards, 2017; Salazar Restrepo and Leite Mariante, 2024; Souza-Rodrigues, 2019; Villoria et al., 2022.

[3] Auerbach, 2022; Battaglini et al., 2022; Boucher and Houndetoungan, 2023; Breza et al., 2020; Debarsy and LeSage, 2022; de Paula et al., 2023; Goldsmith-Pinkham and Imbens, 2013; Halleck Vega and Elhorst, 2015; Han and Lee, 2016; Herstad, 2023; Krisztin and Piribauer, 2022; Lam and Souza, 2020; Lewbel et al., 2023; Manresa, 2016; McCormick and Zheng, 2015; Neumayer and Plümper, 2016; Zhang and Yu, 2018.

Background

Background — priority municipalities

- 86 municipalities **blacklisted as deforestation hotspots**[4]
  - *focused* monitoring & enforcement
  - *spearhead* for interventions & *signal*
- Intervention **revitalized in 2023** development over time

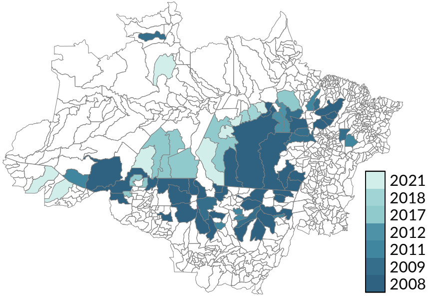


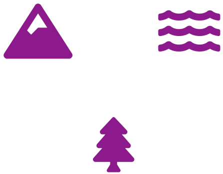
Figure 3: Priority municipalities and the date of designation (from 2008 until 2021).

[4]Studied by Arima et al., 2014; Assunção and Rocha, 2019; Cisneros et al., 2015; de Andrade, 2016.

Framework

Framework — setting

- Consider a **set of regions** $\mathcal{N}$, for which we observe random *responses* Y and *characteristics* X.



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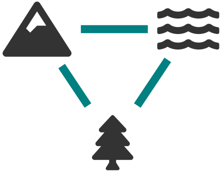

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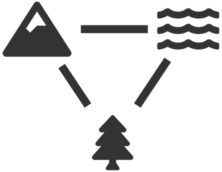
- Regions have a **set of links** $\mathcal{E}$ between them — they are *connected in the network* $\mathcal{G} = \{\mathcal{N}, \mathcal{E}\}$.
- We want to **learn about the relationship**

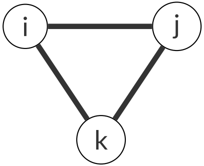
$$Y = f(X, \mathcal{G}) + \varepsilon,$$

- and will build a **hierarchical model** that *imposes structure* on both f and $\mathcal{G}$.



Framework — weighted digraph

Let $\mathscr{G}$ be a graph with a **set of nodes** $\mathscr{N}$ and *links* $\mathscr{E}$, which we allow to be ...



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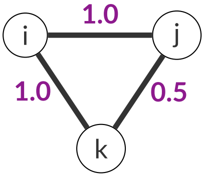
$$g : \mathscr{N} \times \mathscr{N} \mapsto \mathbb{R}^+,$$



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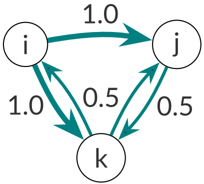

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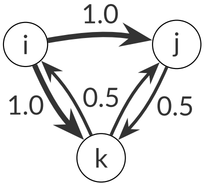

$$\begin{bmatrix} 0 & g_{12} & \cdots & g_{1n} \\ g_{21} & 0 & \cdots & g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & 0 \end{bmatrix}$$

Adjacency matrix

We represent $\mathscr{G}$ with the matrix $\mathbf{G}$, whose entries are $g_{ij} = g(i, j)$, or the **normalized adjacency matrix** $\mathbf{W}$.

Framework — deforestation model

Individual i (in region n) chooses to exert effort $R_i \in \mathbb{R}$ to deforest their plots,[a] **maximizing their utility**:

$$U_i\left(R_i \mid R_{-i}\right) = \left(X_i\tilde{\beta} + W_i X\tilde{\theta} + E_i\right)R_i \; + \; \tilde{\lambda}W_i R\, R_i \; - \; 0.5R_i^2,$$

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where the terms capture ...

- characteristics of i and their links ($W_i X$),
- strategic complementarities.

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where the terms capture ...

- **characteristics** of i and their links ($W_i X$),
- **strategic complementarities**.

The equilibrium best response is *observable*; we observe **regional aggregates** thereof $Y_n = \sum_{i \in n} R_i$. ▸ Details

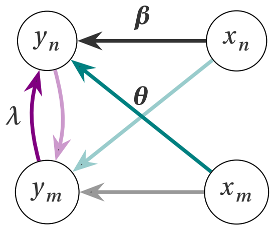
[a] That is, they allocate area between cleared land and forest.

Framework — empirical model

The empirical equivalent is the **linear network model**

$$\mathbf{y} = \lambda\mathbf{W}\mathbf{y} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \mathbf{X}\boldsymbol{\beta} + \mathbf{e},$$

- with **global spillovers** (endogenous), and
- **local spillovers** (contextual) between nodes.



Assumptions

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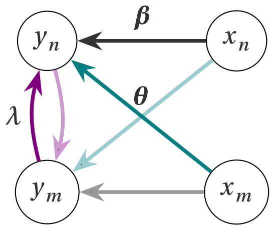


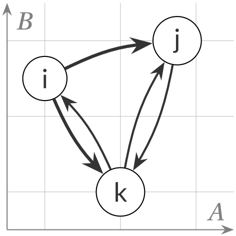
Assumptions

Special cases

Special cases are the *linear-in-means* and *spatial Durbin* models, which **assume a specific W is known**.

Framework — network model

- Assume that **nodes can be embedded** in a **latent space** $(\mathscr{P}, s)$.[a] We have $g_{ij} = s(\Omega, P_i, P_j)$.



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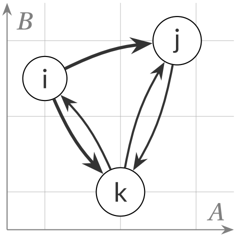
- We can *model links* as decaying in a **metric** d,

$$s := \exp\left\{-\boxed{\delta} \times \boxed{\mathrm{d}(P_i, P_j)}\right\} \quad \forall i \neq j,$$

- where $P_i \in \mathbb{R}^D$ denotes the (latent) *position* of i,
- and the *parameters* control the **rate of decay**.



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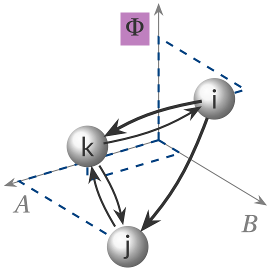

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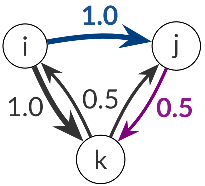
Framework — normalization

In practice, we **normalize** the adjacency matrix to **W**, s.t. spillovers are identified.



$$\mathbf{G} = \begin{bmatrix} 0 & 1.0 & 1.0 \\ 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix},$$

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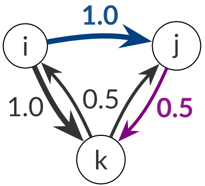
Row normalization

The standard is to transform $\tilde{\mathbf{W}}$ to be **row-stochastic**, such that $\sum_j w_{ij} = 1 \; \forall i$.



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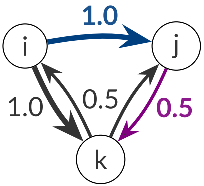
Scalar normalization

We will use **scalar normalization**, such that $w_{ij} = g_{ij} \times \varsigma \; \forall i, j$, in order to **preserve the network structure.** ▸ Which scalar?



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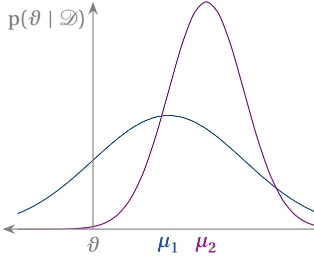
Estimation

Estimation or: How I Learned to Stop Worrying and Love MCMC

I adopt a fully **Bayesian approach**, using *Markov chain Monte Carlo* (MCMC) to obtain posterior samples.

+ nuanced framework to *introduce information*
+ flexible *hierarchical* setups
+ *uncertainty* quantification
– *computational* cost
– put thought into *priors*



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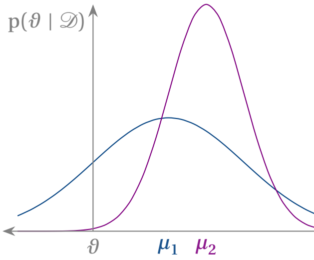

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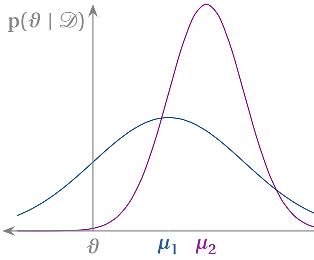


Results

Results — deforestation

- *Average effect* of **blacklisting**
- Deforestation in 808 regions from 2004–2022

$$\mathbf{y}_t = \lambda \mathbf{W}(\cdot)\mathbf{y}_t + \mathbf{X}_t\boldsymbol{\beta} + \mathbf{W}(\cdot)\mathbf{X}_t\boldsymbol{\theta} + \boldsymbol{\mu} + \xi_t + \mathbf{e}_t$$

- Increasingly *flexible networks*
  - *known network* ($\mathbf{0}$, contiguity)
  - latent space *network model*

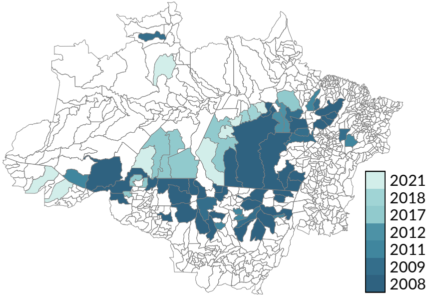
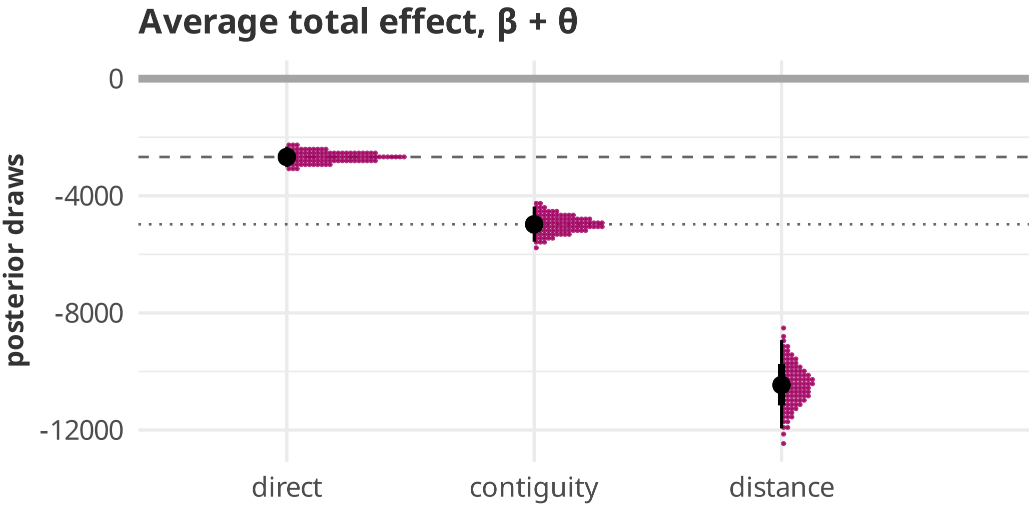


Figure 4: Priority municipalities and the date of designation (from **2008** until 2021).

Direct Effects and Local Spillovers

$$\mathbf{y}_t = \mathbf{X}_t\boldsymbol{\beta} + \mathbf{W}(\delta, \eta_t, \varphi_i)\mathbf{X}_t\boldsymbol{\theta} + \boldsymbol{\mu} + \xi_t + \mathbf{e}_t$$

Results — direct effects & local spillovers



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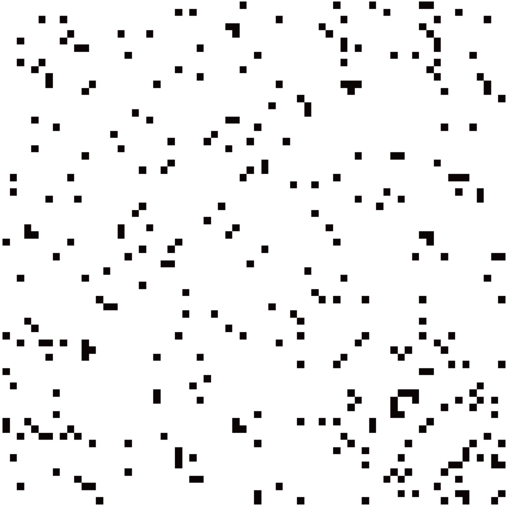
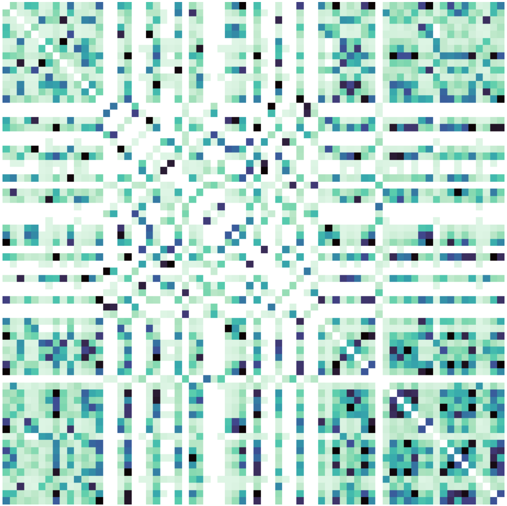
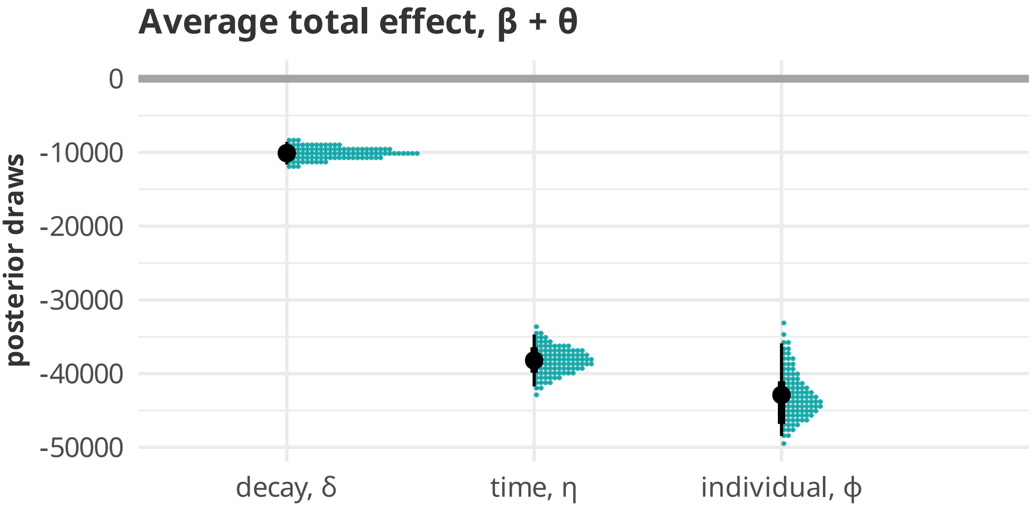
Results — contiguity and distance networks





Results — local spillovers with a network model ‣ All models



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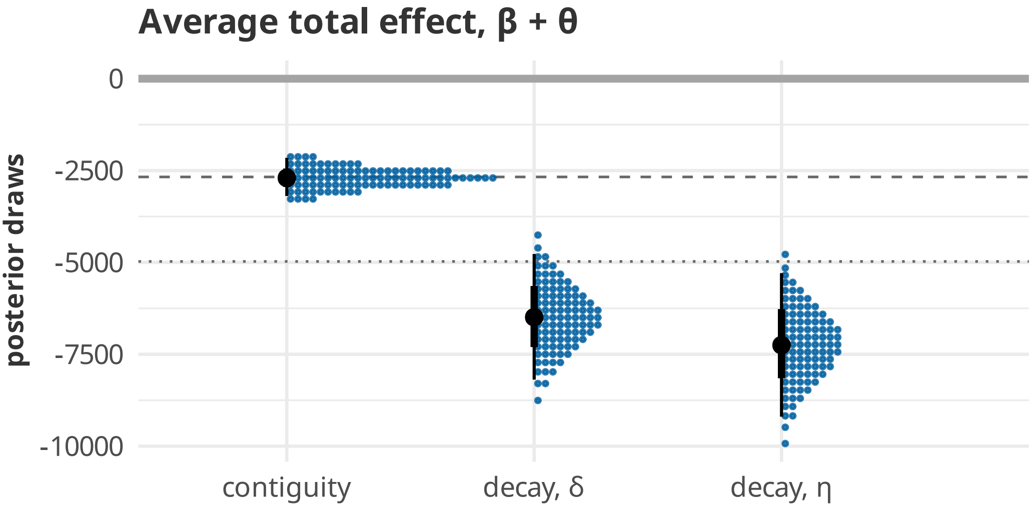
Local and Global Spillovers

$$\mathbf{y}_t = \lambda \mathbf{W}(\delta, \eta_t)\mathbf{y}_t + \mathbf{X}_t \boldsymbol{\beta} + \mathbf{W}(\delta, \eta_t)\mathbf{X}_t \boldsymbol{\theta} + \boldsymbol{\mu} + \xi_t + \mathbf{e}_t$$

Results — local and global spillovers ▸ All models



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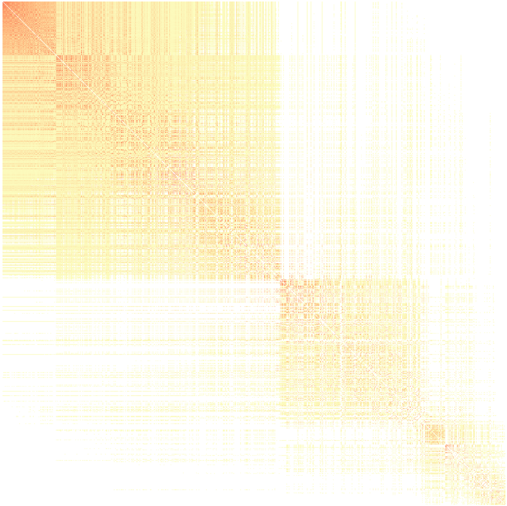
Results — distance and popularity networks



Results — recovering network centrality

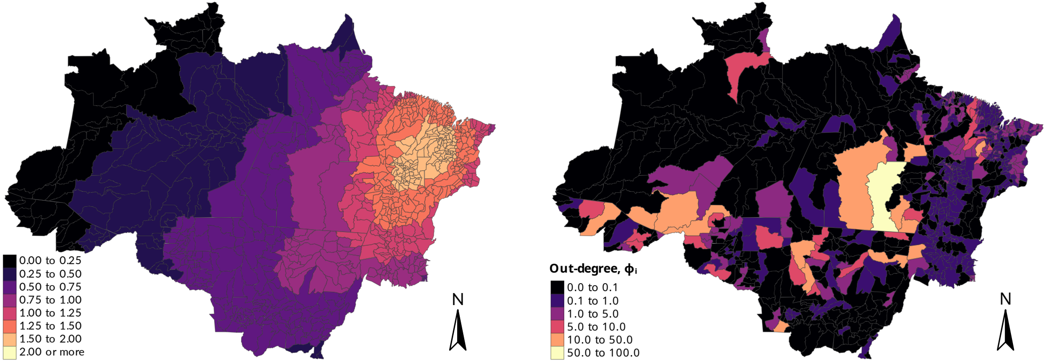


Figure 5: Mean out-degree centrality of $\mathbf{W}(\delta)$ and $\mathbf{W}(\delta, \varphi_i)$. ▸ $\mathbf{W}(\delta, \mathbf{P})$

Results — simulation

1. US states, **asymmetry** from population ‣ Results
2. US states, **weak prior** position ‣ Results
3. Erdős–Rényi graph, latent space **approximation** ‣ Results

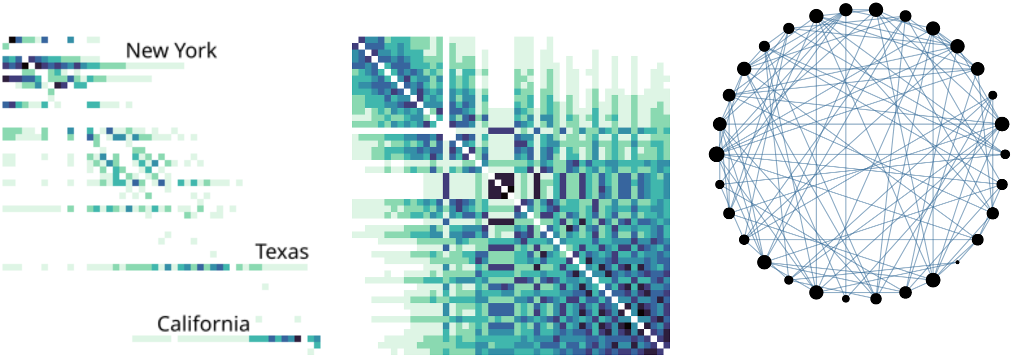


Figure 6: Illustration of the asymmetric network between US states, the centroid network, and the Erdős–Rényi graph.

Conclusion

Conclusion

- **Joint model** of *spillovers* and *networks*
  - leverages *data* and *structure*
  - yields *deeper insights* into spillovers
  - moderate to pronounced *computational costs*
- **Amazon deforestation** spillovers
  - *underestimated* with restrictive models
  - confounding from *endogenous spillovers*
  - *network structure* to utilize

Thank you for your attention!

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References i

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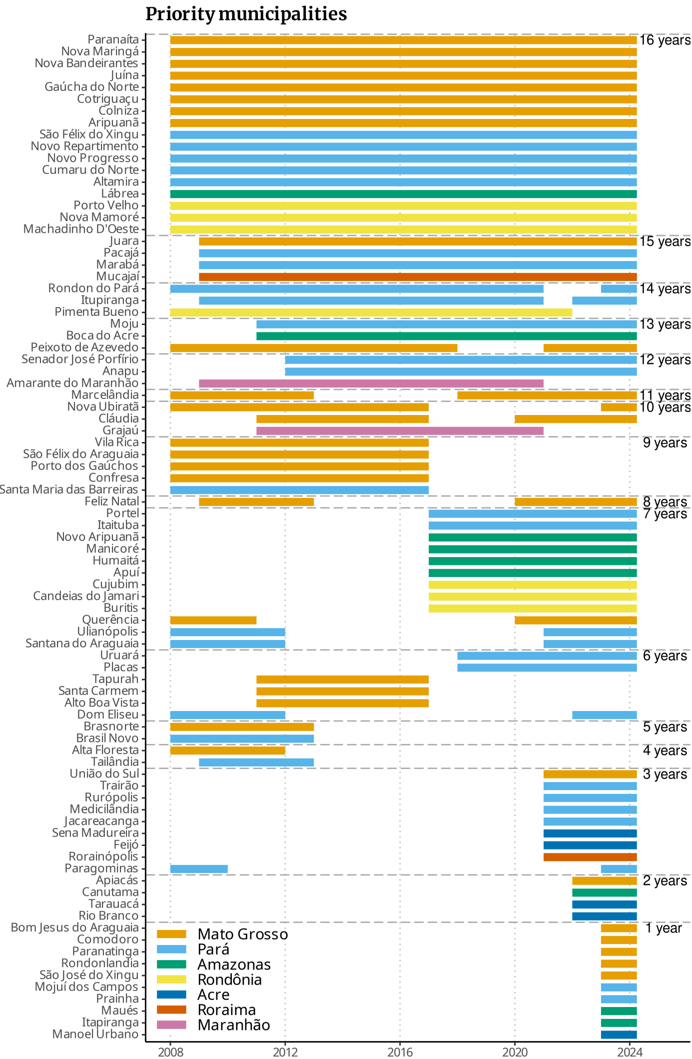
Priority municipalities
Paranaíta
Nova Maringá
Nova Bandeirantes
Juína
Gaúcha do Norte
Cotriguaçu
Colniza
Aripuanã
São Félix do Xingu
Novo Repartimento
Novo Progresso
Cumaru do Norte
Altamira
Lábrea
Porto Velho
Nova Mamoré
Machadinho D'Oeste
Juara
Pacajá
Marabá
Mucajaí
Rondon do Pará
Itupiranga
Pimenta Bueno
Moju
Boca do Acre
Peixoto de Azevedo
Senador José Porfírio
Anapu
Amarante do Maranhão
Marcelândia
Nova Ubiratã
Cláudia
Grajaú
Vila Rica
São Félix do Araguaia
Porto dos Gaúchos
Confresa
Santa Maria das Barreiras
Feliz Natal
Portel
Itaituba
Novo Aripuanã
Manicoré
Humaitá
Apuí
Cujubim
Candeias do Jamari
Buritis
Querência
Ulianópolis
Santana do Araguaia
Uruará
Placas
Tapurah
Santa Carmem
Alto Boa Vista
Dom Eliseu
Brasnorte
Brasil Novo
Alta Floresta
Tailândia
União do Sul
Trairão
Rurópolis
Medicilândia
Jacareacanga
Sena Madureira
Feijó
Rorainópolis
Paragominas
Apiacás
Canutama
Tarauacá
Rio Branco
Bom Jesus do Araguaia
Comodoro
Paranatinga
Rondonlandia
São José do Xingu
Mojuí dos Campos
Prainha
Maués
Itapiranga
Manoel Urbano
16 years
15 years
14 years
13 years
12 years
11 years
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9 years
8 years
7 years
6 years
5 years
4 years
3 years
2 years
1 year
Mato Grosso
Pará
Amazonas
Rondônia
Acre
Roraima
Maranhão
2008
2012
2016
2020
2024

Intervention erosion — fine status

Back

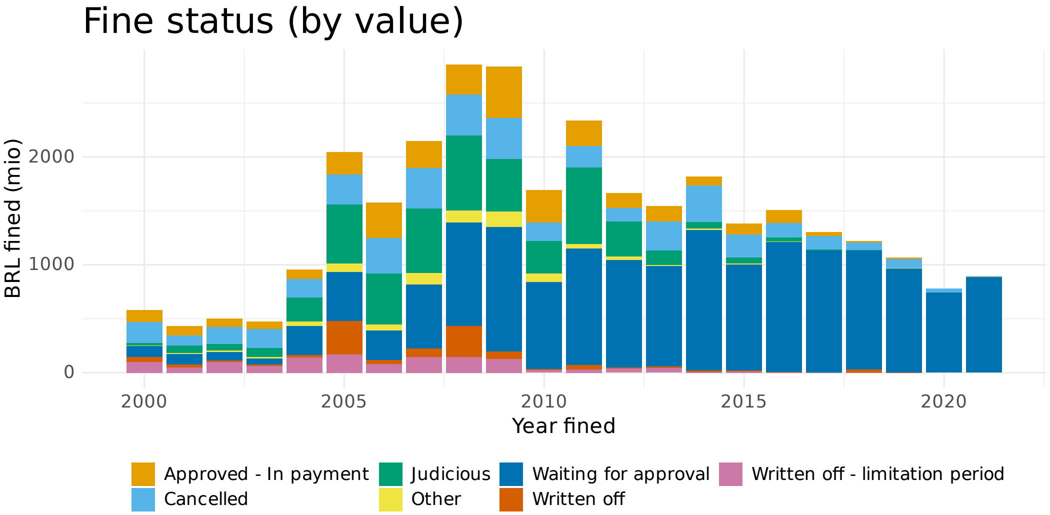


Figure 7: Status of forest-related environmental fines (Kuschnig et al., 2023).

Intervention erosion — fine status

Back

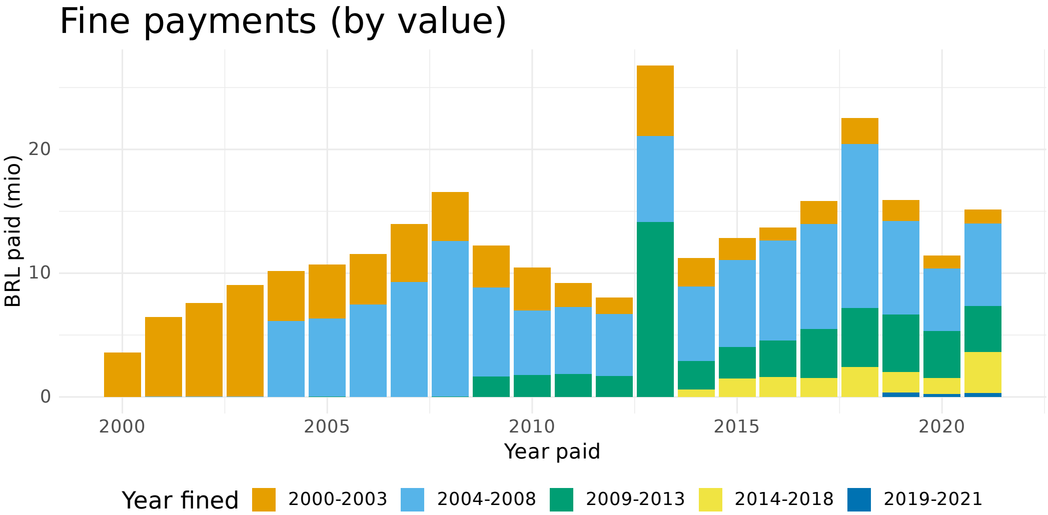


Figure 7: Status of forest-related environmental fines (Kuschnig et al., 2023).

Microfoundations

Back

The individual equilibrium best response is[5]

$$R_i = \tilde{\lambda} W_i R + X_i \tilde{\beta} + W_i X \tilde{\theta} + E_i$$

We observe **aggregates**; **region-internal** networks are absorbed:

$$\begin{matrix} R_{m1} \\ R_{m2} \\ R_{n1} \\ R_{n2} \\ R_{o1} \\ \vdots \end{matrix} = \tilde{\lambda} \begin{bmatrix} 0 & w_{m1m2} & w_{m1n1} & w_{m1n2} & \dots \\ w_{m2m1} & 0 & w_{m2n1} & w_{m2n2} & \dots \\ w_{n1m1} & w_{n1m2} & 0 & w_{n1n2} & \dots \\ w_{n2m1} & w_{n2m2} & w_{n2n1} & 0 & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} R + \dots$$

[5] Subject to conditions (see Blume et al., 2015) including stationarity of the network.

Linear network model ◂ Back

For the network, we require (i) stationarity of the network filter $\mathbf{I} - \lambda\mathbf{W}$, (ii) spillover effects that do not cancel out $\lambda\beta + \theta \neq 0$, (iii) network asymmetry, i.e., $\operatorname{diag}\left(\mathbf{W}^2\right) \not\propto \iota$ (see de Paula et al., 2023). If, additionally, $\lambda > 0$ the parameters are globally identified.

Due to the presence of the endogenous spillover and individual fixed effects, we require **strict exogeneity** to consistently estimate the parameters. (That is, there is no autocorrelation of Y and errors are uncorrelated with the network structure.)

The endogenous spillover gives rise to a multiplier effect, but is often suppressed due to the unloved assumptions required; if it is suppressed, however, other network effects may be confounded.

Normalization for interpretation ◂ Back

Consider a contextual model,

$$\mathbf{z} = \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \mathbf{X}\boldsymbol{\beta} + \mathbf{e}.$$

The partial effect of a regressor k is given by

$$\frac{\partial \mathbf{z}}{\partial \mathbf{x}_k} = \mathbf{I}\beta_k + \mathbf{W}\theta_k.$$

We usually associate parameters with the **average partial effect** (APE),[a] and can interpret β_k as the *direct APE*, and can fix $\boldsymbol{\theta}$ at the *indirect APE* by setting $\mathbf{W}$ such that $\sum_i \sum_j w_{ij} = N$.

[a] An alternative, when some agents are not linked in the network, is the average partial effect for all linked agents.

For the full linear network model, let $\mathbf{z}$ be a latent quantity that goes the network filter.

The same reasoning holds for the autoregressive parameter λ.

The scaling factor is

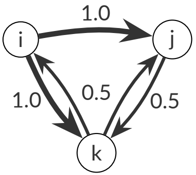
$$\frac{N}{\sum_i \sum_j g_{ij}}.$$

Normalization for estimation ◂ Back

Consider a **network autoregression**

$$\mathbf{y} = (\mathbf{I} - \lambda \mathbf{W}\mathbf{y})^{-1}\, \mathbf{z}.$$

We need to ensure invertibility and stability.



Theorem

Let $\mathbf{I}$ denote the identity matrix, and α be a real scalar. Then $\mathbf{I} - \alpha\mathbf{A}$ is stationary for $\alpha \in \left(-\rho_{\mathbf{A}}, \rho_{\mathbf{A}}\right)$, where $\rho_{\mathbf{A}}$ denotes the spectral radius of $\mathbf{A}$.

Row-normalization distorts, e.g., the *eigencentrality*

EC	i	j	k
$\mathbf{G}$	2^{-1}	6^{-1}	3^{-1}
$\tilde{\mathbf{W}}$	3^{-1}	3^{-1}	3^{-1}.

For estimation, we normalize the autoregressive term with the **inverse spectral radius**. This relates the parameter $\lambda^s \in (-1, 1)$ to the *dominant eigenvector* of the network.

By the spectral radius being the infimum norm, λ^s is *at most* the average partial effect.

A Beta prior ...

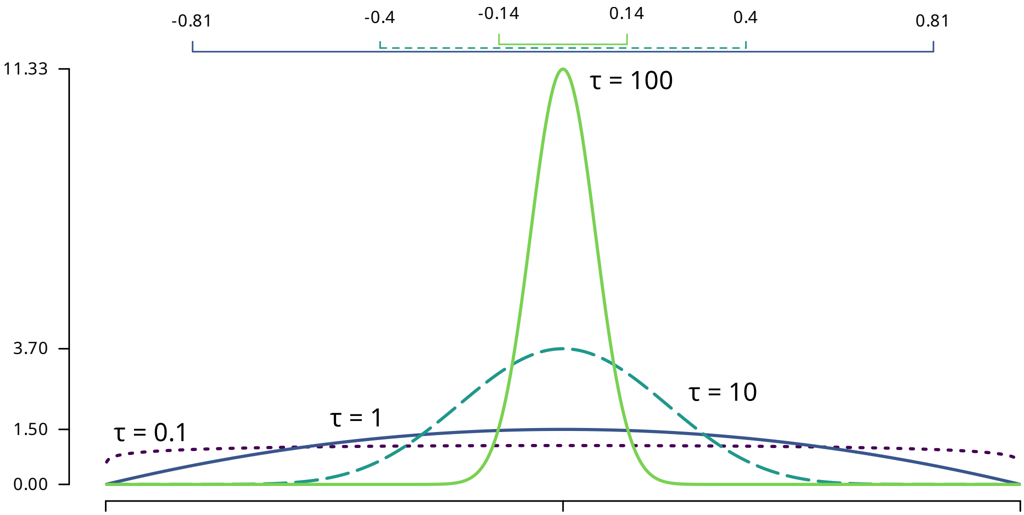


Figure 8: Scaled $\mathrm{Beta}(1+\tau, 1+\tau)$ densities with increasing weight, τ.

...with a Gamma mixing distribution

Back

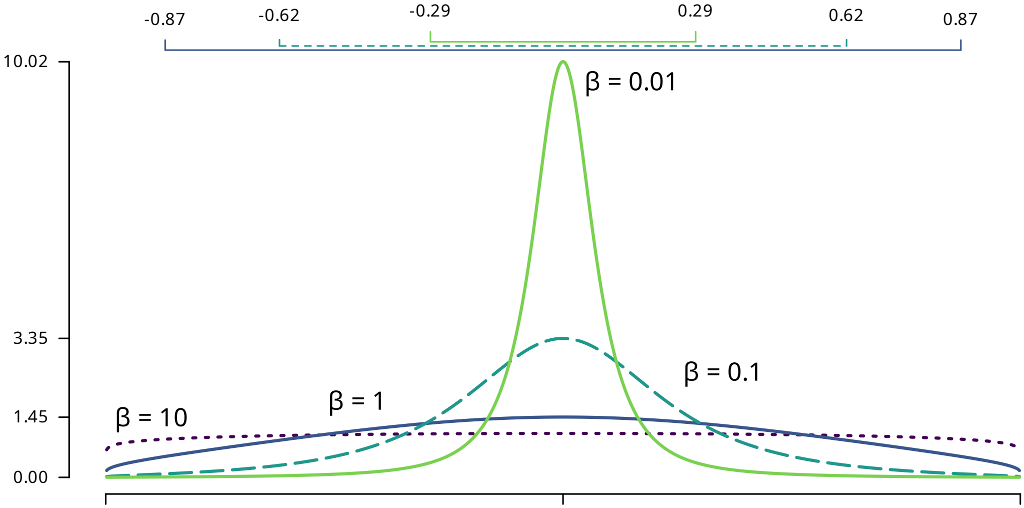


Figure 9: Scaled $\mathrm{Beta}(1+\tau, 1+\tau), \tau \sim \mathrm{Exp}(\beta)$ with increasing weight, $\mathbb{E}[\tau] = 1/\beta$.

Centroids — Legal Amazon

◂ Back

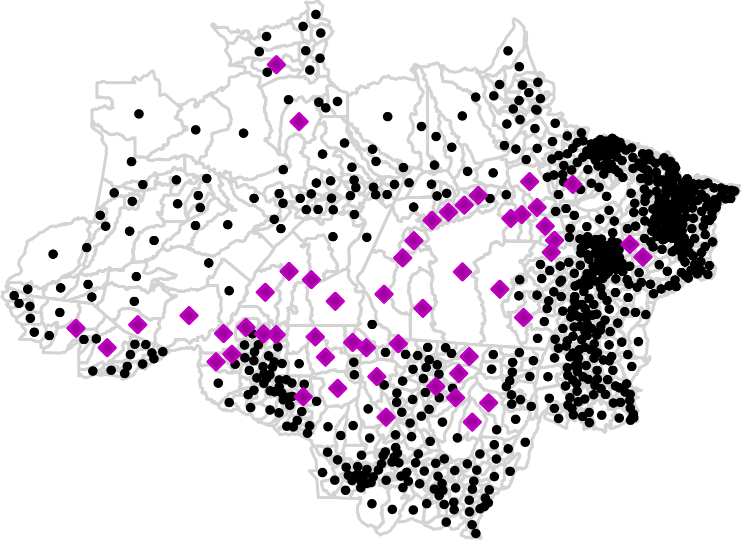


Figure 10: Centroids of the municipalities in the Legal Amazon. Priority municipalities are marked in **purple**.

Sampling ◂ Back

- **Gibbs** sampler
  0. Let i be zero and set an initial value $\tilde{\beta}_i$
  1. Draw σ^2_{i+1} from $p(\sigma^2 \mid \tilde{\beta}, \cdot)$
  2. Draw $\tilde{\beta}_{i+1}$ from $p(\tilde{\beta} \mid \sigma^2_{i+1}, \cdot)$
  3. Increment i and go to Step 1 if more samples are desired.
- **Metropolis-Hastings** step (for δ)
  0. Let i be zero and set an initial value δ_i
  1. Propose a value for δ^* using $\mathcal{Q}(\delta^* \mid \delta_i)$
  2. Set δ_{i+1} to δ^* with probability $\alpha(\delta^* \mid \delta_i)$
  3. Increment i and go to Step 1 if more samples are desired.

The acceptance probability α is the ratio of the conditional posteriors and proposal density Q evaluated at the current and proposed values. When $\dim(\delta)$ is large, we loop over blocks of its elements to improve mixing.

Jacobian determinant
◂ Back

In standard models, we'd compute the **spectral decomposition** of $\mathbf{W}$ **once** and compute the determinant using $\mathbf{W}$'s eigenvalues (ω_i), as

$$\ln|\mathbf{I} - \lambda\mathbf{W}| = \sum_{i=1}^{N} \ln\left(1 - \lambda\omega_i\right).$$

However, our $\mathbf{W}$ is **mutable**, and computing eigenvalues for every draw of δ is prohibitive at $\mathcal{O}(N^3)$ complexity.

With limited parameters for the network structure, I propose

- the **Gaussian process approximation**

$$|\mathbf{S}(\lambda, \cdot)| \approx \mathrm{GP}\left(\mu(\lambda, \cdot), \mathbf{\Sigma}(\lambda, \cdot)\right),$$

- with *Arnoldi / Lanczos* for normalization of $\mathbf{W}$.

Gaussian process approximation ◂ Back

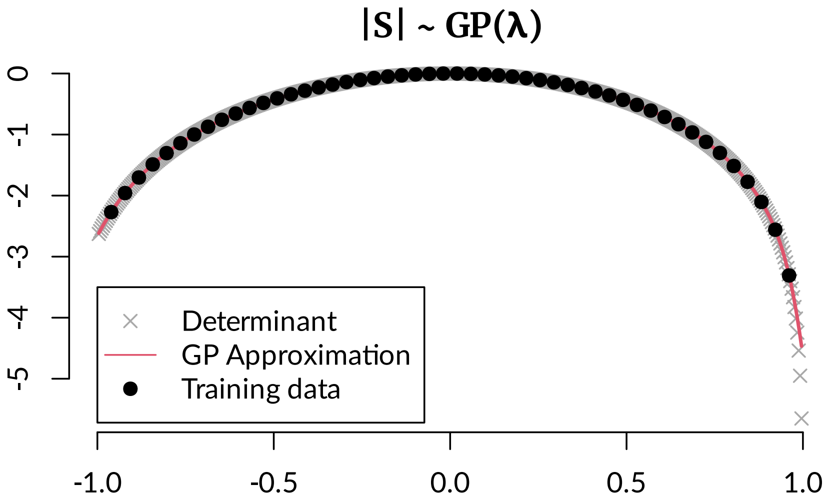


Figure 11: GP approximation to $|\mathbf{S}(\lambda)|$ using 50 training samples. Distances are between $N = 100$ locations with Uniform random coordinates.

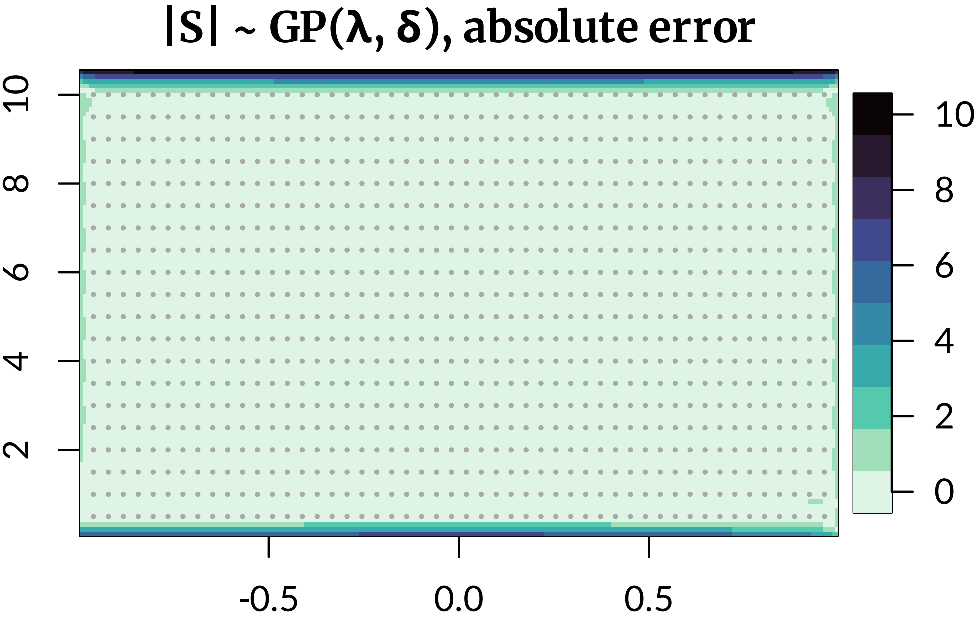
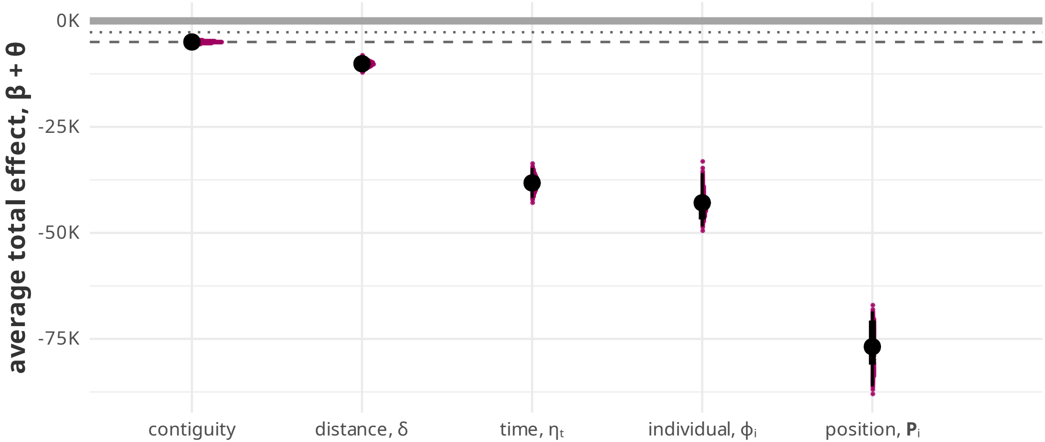


Figure 12: GP approximation to $|\mathbf{S}(\lambda, \delta)|$ using 50×20 training samples.

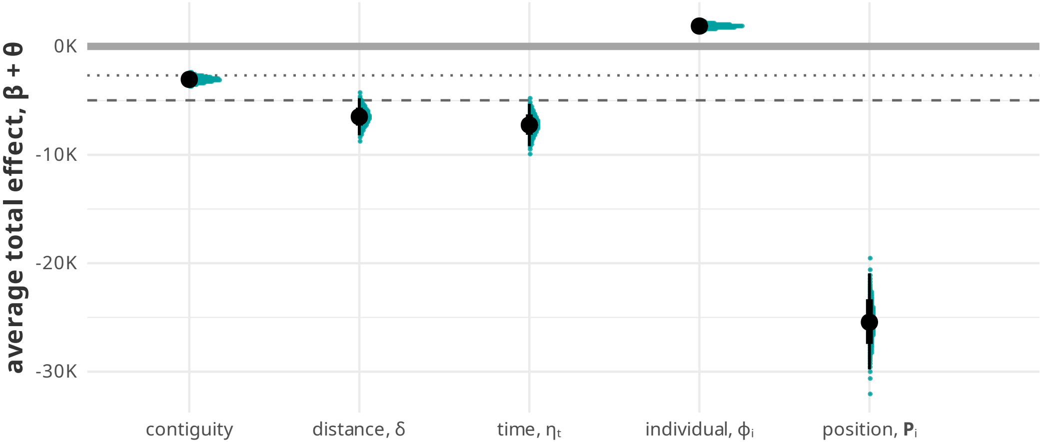
Results — local spillovers

Back



Results — local and global spillovers

◂ Back



Outdegrees by treatment status

◂ Back

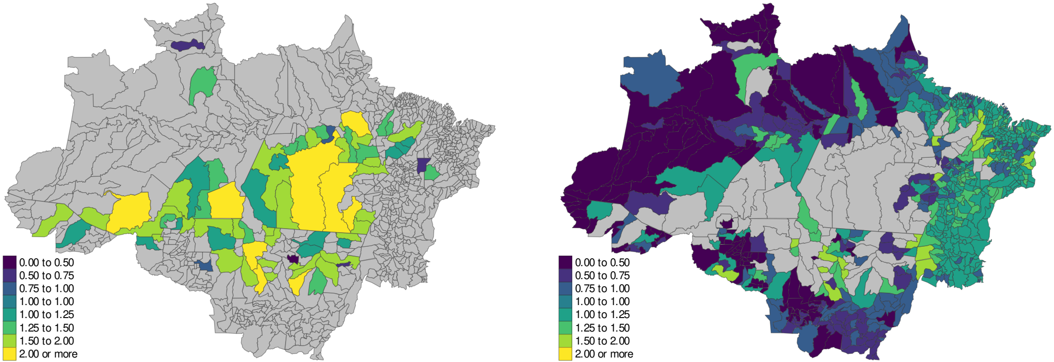


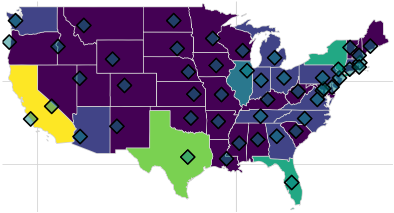
Figure 13: Relative outdegrees of priority (left) and other (right) municipalities as obtained from the full model, using $\mathbf{W}(\delta, \mathbf{P})$.

Simulation — US population ◂ Back

The network is determined as

$$g_{ij} = \exp\left\{-\delta_i \times d\left(\mathbf{p}_i, \mathbf{p}_j\right)\right\},$$

where **popularity** δ_i depends on a state's population, and $\mathbf{p}_i$ its **population center**.



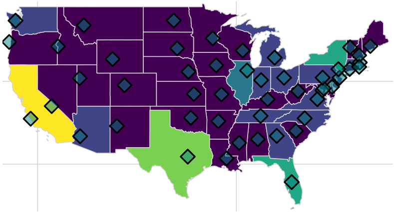
State population and its centers.

Simulation — US population ◂ Back

The network is determined as

$$g_{ij} = \exp\left\{-\delta_i \times \mathrm{d}\left(\mathbf{p}_i, \mathbf{p}_j\right)\right\},$$

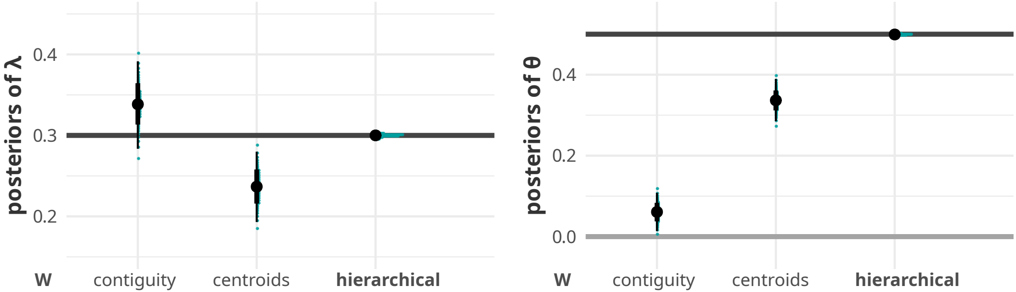
where **popularity** δ_i depends on a state's population, and $\mathbf{p}_i$ its **population center**.

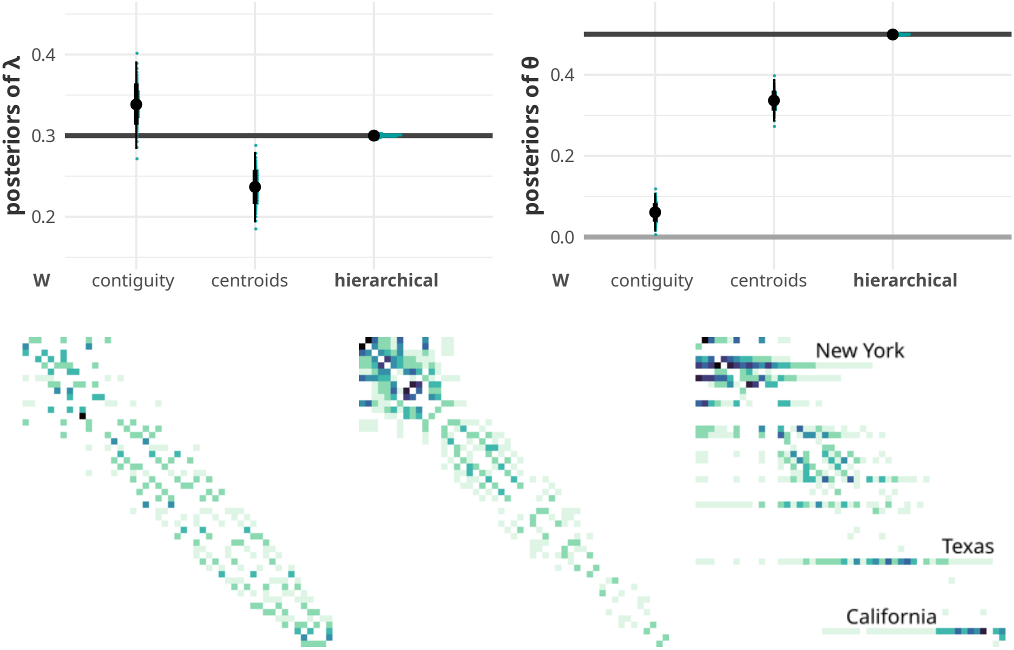


State population and its centers.

We'll start by estimating λ, θ in a long panel ($T = 50$) using

- **contiguity** between states,
- distance-decay between **centroids**, and
- the *true model* of **location & popularity**.


posteriors of λ
0.4
0.3
0.2
W
contiguity
centroids
hierarchical
posteriors of θ
0.4
0.2
0.0
W
contiguity
centroids
hierarchical



W based on **contiguity**, **distance-decay**, and the **true network**.

Simulation — US centroids ◂ Back

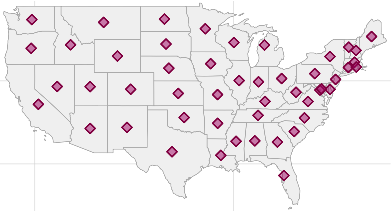
The network is determined as

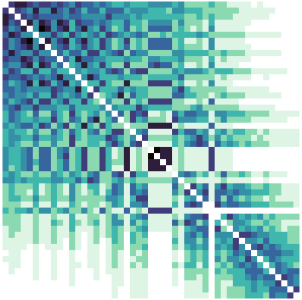
$$g_{ij} = \exp\left\{-\delta \times \mathrm{d}\left(\mathbf{p}_i, \mathbf{p}_j\right)\right\},$$

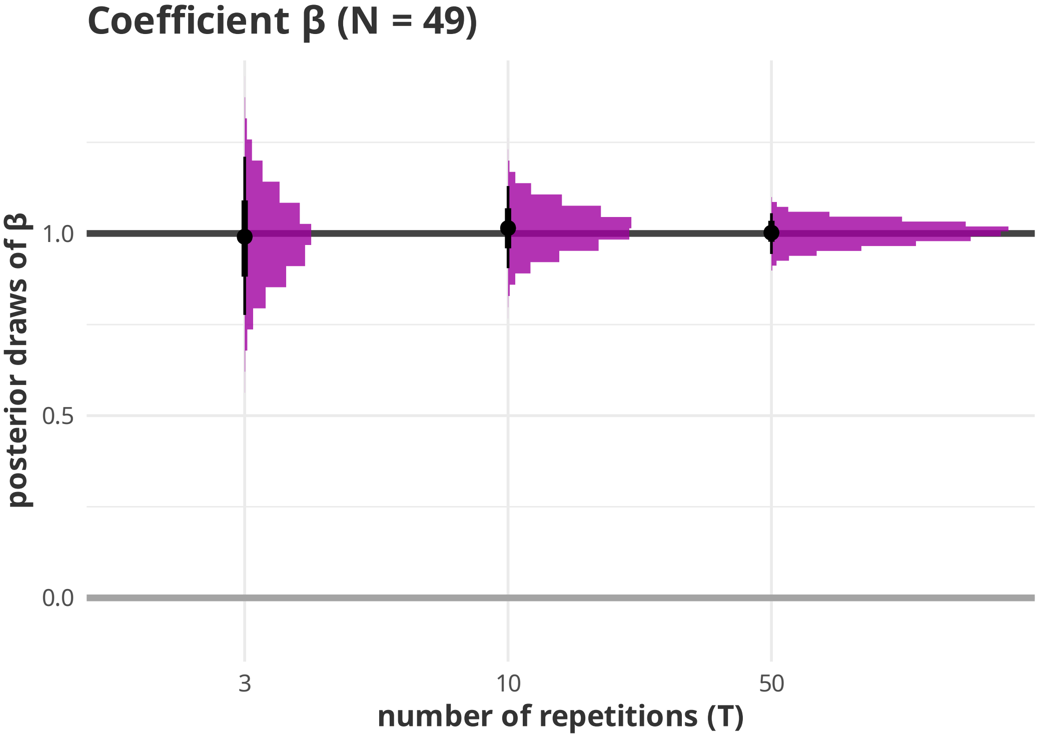
where $\mathbf{p}_i$ are **centroids**, and

- the network is rather **dense**,
- I initialize the sampler at **random locations** to illustrate convergence.

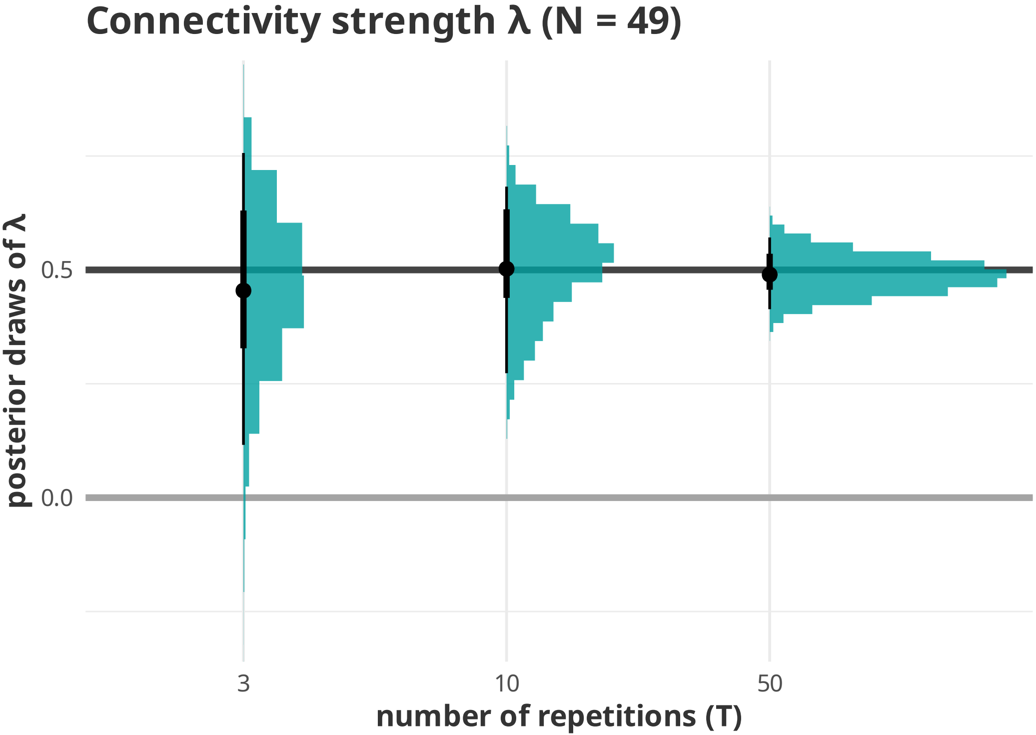
We'll have a look at **posteriors of λ** for multiple simulations after a **short burn-in** of 1,000 draws.







Coefficient β (N = 49)
posterior draws of β
1.0
0.5
0.0
3
10
50
number of repetitions (T)


Connectivity strength λ (N = 49)
posterior draws of λ
0.5
0.0
3
10
50
number of repetitions (T)

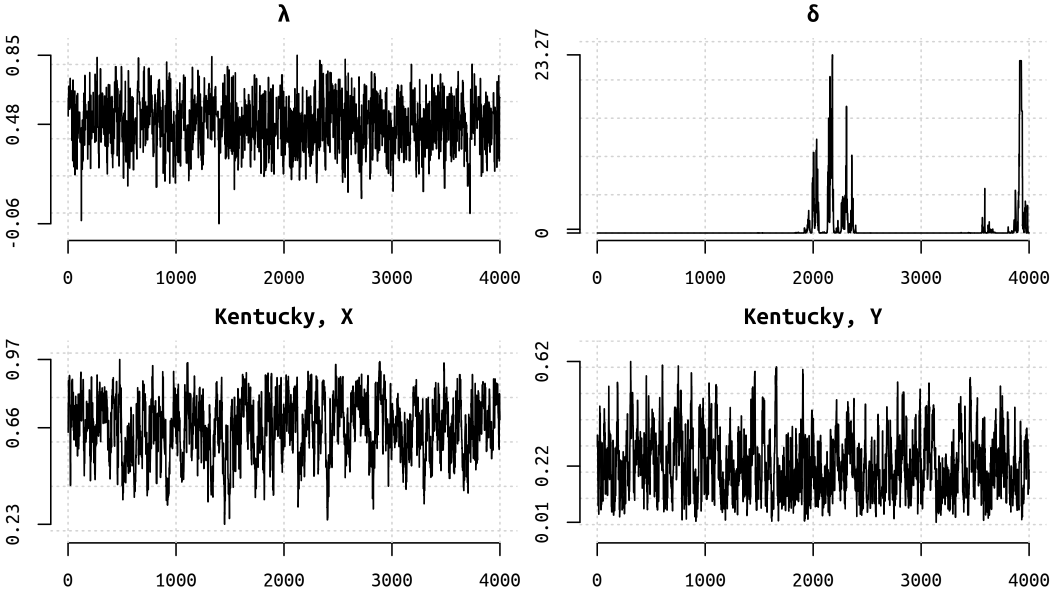


Figure 14: Traceplots for the posterior draws of λ, δ (note the poor mixing), and the (scaled) coordinates of Kentucky based on $T = 3$ network observations.

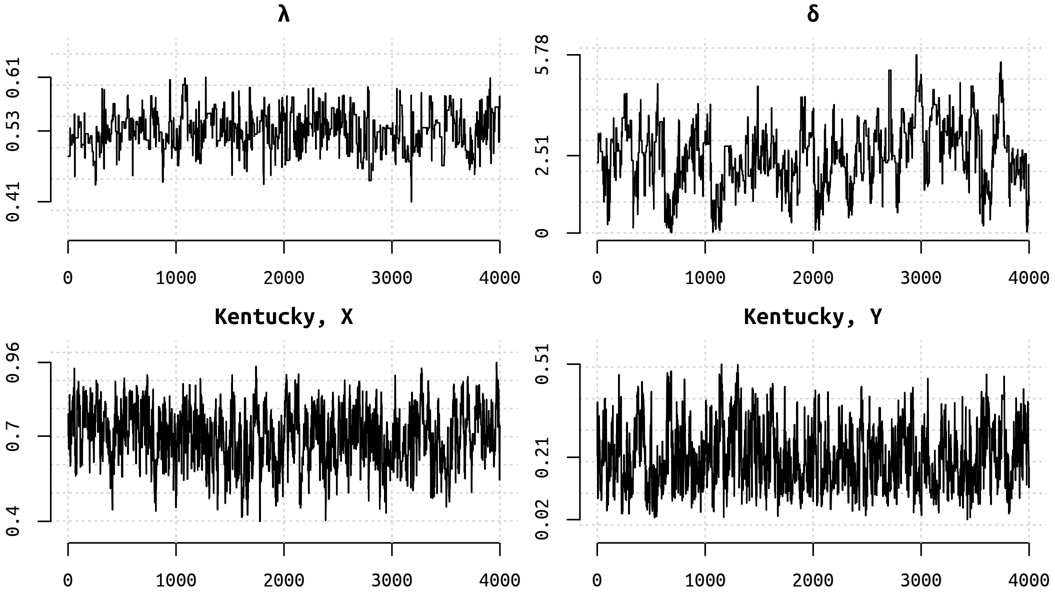


Figure 15: Traceplots for the posterior draws based on $T = 50$ network observations. Note the improved mixing behavior of δ. Back

Simulation — the Erdős–Rényi graph

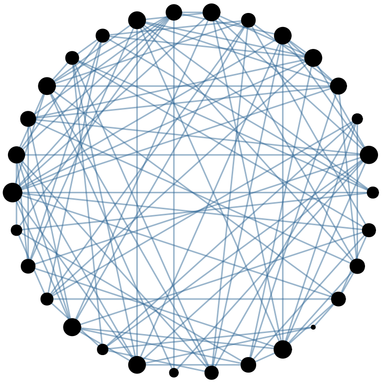
Back

The graph ($N \in \{30, 50\}$) is determined as

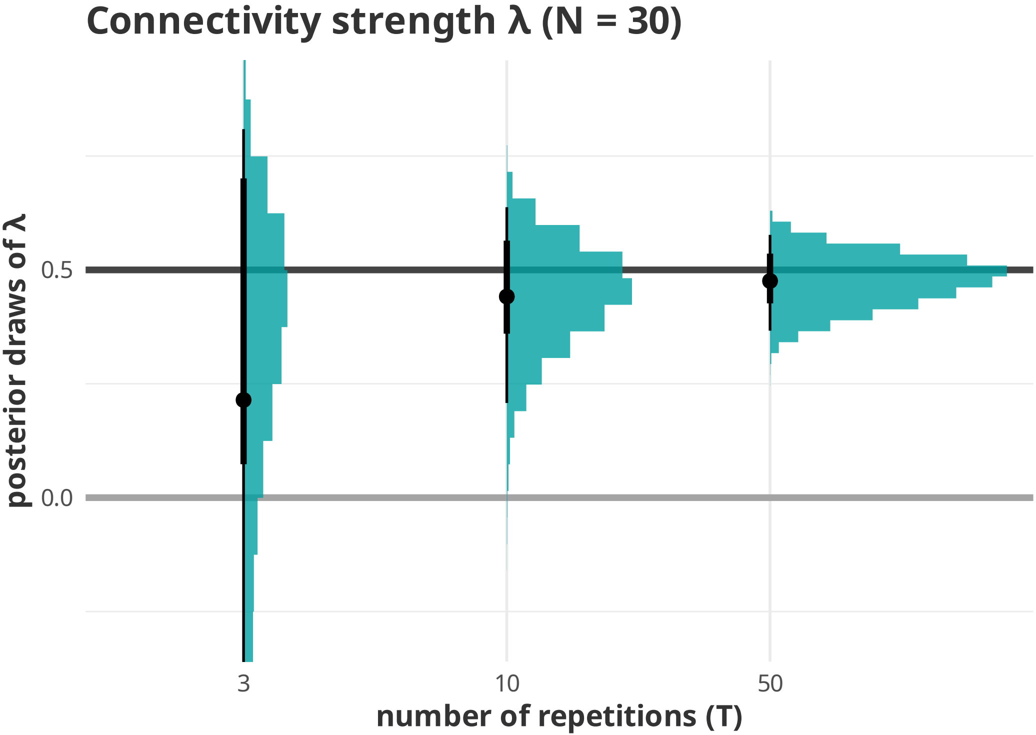
$$g_{ij} = \begin{cases} 1 \text{ with probability } 0.25, \\ 0 \text{ otherwise.} \end{cases}$$

Our model uses the same **distance-decay** specification as before, but no true embedding exists.

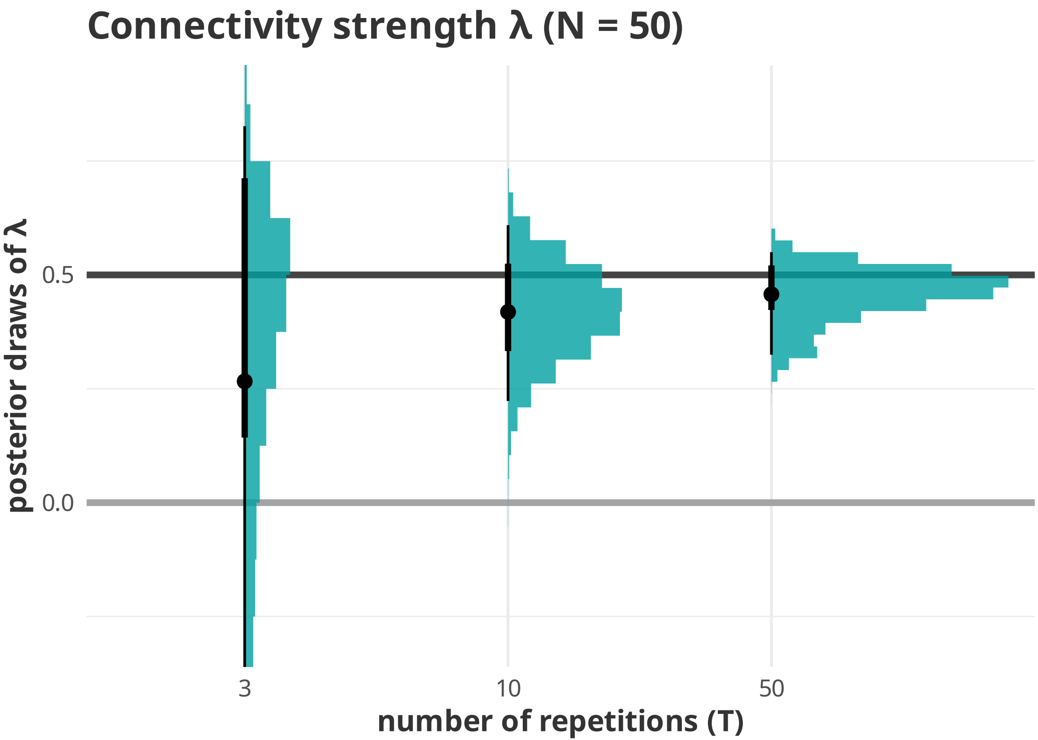
To **highlight convergence**, we'll have a look at *posteriors of* λ for multiple simulations after a **short burn-in** of 1,000 draws.



A realization of $\mathcal{G}(30, 0.25)$.



Connectivity strength λ (N = 30)
posterior draws of λ
0.5
0.0
3
10
50
number of repetitions (T)


Connectivity strength λ (N = 50)
posterior draws of λ
0.5
0.0
3
10
50
number of repetitions (T)